arXiv:nucl-th/9507043v1 28 Jul 1995

Interaction of η-meson with light nuclei

V. B. Belyaev
Joint Institute for Nuclear Research, Dubna, 141980, Russia

S. A. Rakityansky*, S. A. Sofianos, and M. Braun
Physics Department, University of South Africa, P.O.Box 392, Pretoria 0001, South Africa

W. Sandhas
Physikalisches Institut, Universitắt Bonn, D-53115 Bonn, Germany

(February 9, 2008)

Abstract

A microscopic treatment of η–nucleus scattering is presented. When applying the underlying exact integral equations, the excitation of the target are neglected, and an input ηN amplitude is chosen which reproduces the $S_{11}(1535)$ resonance. It is shown that the η–nucleus scattering lengths are quite sensitive to the ηN parameters and the nuclear wave functions. For a special choice of the parameters an η^4He quasi–bound state occurs.

I. INTRODUCTION

There are at least three main reasons which motivate studies concerning η–nucleus systems.

The first one is of a fundamental character and is related to the possibility of studying the quark structure of η–mesons and the nucleon $S_{11}(1535)$–resonance, and the role played by the strange quarks in these systems. Recently, in experiments with $N\overline{N}$ annihilation into ϕ and ω channels [1], it was established that there is a strong polarization of the strange sea–quarks in nucleons. It is, therefore, interesting to investigate possible manifestation of this phenomenon also in η–nucleus systems.

The second reason concerns a pure nuclear problem, namely, the possible formation of η – nucleus bound states. It was argued by Haider and Liu [2] that, due to the rather strong attraction in the ηN–interaction at low energies, the η–meson, when immersed in the nucleus, might create a system which in some respects is analogous to a hypernucleus.

---

*Permanent address: Joint Institute for Nuclear Research, Dubna, 141980, Russia

Estimations in [2] indicated that such a possibiltity exists for nuclei with atomic number $A \geq 12$.

Thirdly, Charge Symmetry Breaking (CSB) effects can be studied in processes involving η–production. Indeed, CSB was experimentally observed at Saturne [3] in the reaction

$$d + d \longrightarrow {}^4He + \pi^0, \tag{1}$$

near the threshold of η–production. It was found that the production of pions was much greater than the CSB expected from electromagnetic interaction. The experimental cross–section is equal to 0.97 pb/sr (at $E_d = 1100$ MeV), while the one obtained from electromagnetic considerations is 0.003 pb/sr [4], the latter estimate being made at 600 MeV.

A possible explanation of CSB in the above reaction is that it can be attributed to the $\eta - \pi^0$ mixing, i.e., the process (1) can proceed via intermediate production of a pure η^0 meson state with isospin $I = 0$,

$$d + d \longrightarrow {}^4He + \eta^0, \tag{2}$$

which is an allowed reaction. The state $|\eta^0\rangle$ can be expressed as a linear combination of the physical ones, $|\eta\rangle$ and $|\pi^0\rangle$,

$$|\eta^0\rangle = \frac{1}{\sqrt{1+\lambda^2}}(\,|\eta\rangle + \lambda|\pi^0\rangle\,), \tag{3}$$

where the parameter λ characterises the $\eta - \pi^0$ mixing. From Eq. (3) it follows that λ is the proportionality constant between the amplitudes [5]

$$f(dd \to \alpha\pi^0) \sim \lambda f(dd \to \alpha\eta). \tag{4}$$

The process (1), therefore, is no longer forbidden and one can expect a cross section much larger than the electromagnetic one.
Based on the above ideas, the authors of Ref. [6] obtained 0.12 pb/sr for the cross–section of the process (1) as compared to the experimental value of 0.97 pb/sr. To explain this discrepancy, fully microscopic few–body calculations, which take into account the $\eta - \pi^0$ mixing, should be performed.

We would also like to mention the possibility of studying processes of the type

$$d + d \longrightarrow {}^4He^*\,(\mathcal{J}^P, 0) + \pi^0. \tag{5}$$

Here, ${}^4He^*\,(\mathcal{J}^P, 0)$ denotes excited states of ${}^4He$ nucleus with isotopic spin $I = 0$ and different angular momenta and parities. These reactions might provide us with large cross sections due to the extended size of the excicted ${}^4He$ nucleus.

Having the above in mind, and in order to get some insight into the physics of η–nucleus systems, we undertook microscopic investigations concerning the low–energy behaviour of the η–nucleus elastic scattering amplitude and the positions of poles of the corresponding t-matrices in the complex k–plane. Another issue addressed is up to what extent the attraction of the two–body ηN interaction should be enhanced in order to produce quasi-bound states in the η–nucleus system.

II. THE METHOD

Our method is based on the so–called Finite–Rank–Approximation (FRA) of the Hamiltonian proposed as an alternative to the multiple scattering theory of pion-nucleus interaction [7]. We, therefore, start by outlining this approach.

Consider the scattering of an η-meson from a nucleus of atomic number A. The total Hamiltonian of this system is

$$H = H_0 + V + H_A, \tag{6}$$

where H_0 is the kinetic energy operator (free Hamiltonian) of the η–nucleus motion , $V = V_1 + V_2 + \cdots + V_A$ is the sum of ηN–potentials, and H_A is the total Hamiltonian of the nucleus. Introducing the Green function

$$G_A(z) = \frac{1}{z - H_0 - H_A}, \tag{7}$$

we obtain the following equation for the transition operator

$$T(z) = V + VG_A(z)T(z). \tag{8}$$

For further developments, we introduce the (auxiliary) transition operator $T^0(z)$ via

$$T^0(z) = V + VG_0(z)T^0(z), \tag{9}$$

where

$$G_0(z) = \frac{1}{z - H_0} \tag{10}$$

is the free Green function. From the definition of $G_0(z)$ and V, it is clear that T^0 describes the scattering of mesons by a nucleus in which the position of the nucleons is fixed. The operator T^0 differs from the usual fixed center t-matrix in two respects. Firstly, Eq. (9) contains the kinetic energy operator H_0 describing the motion of the η–meson with respect to the center of mass of the target, secondly, the energy argument of T^0 is taken to be that of the total energy z of the system.

Using the resolvent equation

$$G_A(z) = G_0(z) + G_0(z)H_AG_A(z), \tag{11}$$

we easily infer from (8) and (9)

$$T(z) = T^0(z) + T^0(z)G_0(z)H_AG_A(z)T(z), \tag{12}$$

a form suitable for the low–energy approximation on which our approach is essentially based. The spectral decomposition of the nuclear Hamiltonian H_A reads

$$H_A = \sum_n \mathcal{E}_n^A |\Psi_n^A\rangle\langle\Psi_n^A| + \int E|\Psi_E^A\rangle\langle\Psi_E^A|\, dE\,, \tag{13}$$

where $|\Psi_n^A\rangle$ are the bound–state eigenfuntions of H_A with $\mathcal{E}_n$ being the correponding energies. Since we are interested in processes at energies far below the excited states or break–up thresholds of the nuclei, we can neglect all contributions to (13) except the ground–state part. That is, we use the approximation

$$H_A \approx \mathcal{E}_0^A |\Psi_0^A >< \Psi_0^A|. \tag{14}$$

Inserting Eq. (14) into Eq. (12) and sandwiching with $|\Psi_0^A\rangle$, we obtain the Lippmann–Schwinger–type equation

$$T(\vec{k}', \vec{k}; z) = T^0(\vec{k}', \vec{k}; z) + \mathcal{E}_0^A \int \frac{d^3q}{(2\pi)^3} \frac{T^0(\vec{k}', \vec{q}; z)}{(z - \dfrac{q^2}{2\mu})(z - \mathcal{E}_0 - \dfrac{q^2}{2\mu})} T(\vec{q}, \vec{k}; z), \tag{15}$$

where

$$T^0(\vec{k}', \vec{k}; z) = \int d^{3(A-1)} r |\Psi_0^A(\vec{r})|^2 T^0(\vec{k}', \vec{k}; \vec{r}; z), \tag{16}$$

Here $\vec{r}$ represents all nuclear Jacobi coordinates.

From a practical point of view it is convenient to rewrite Eq. (9) by using the Faddeev-type decomposition

$$T^0(z) = \sum_{i=1}^{A} T_i^0(z).$$

Introducing the operators

$$t_i(z) = V_i + V_i \frac{1}{z - H_0} t_i(z),$$

we finally get for the Faddeev components T_i^0 the following system of integral equations

$$\begin{aligned} T_i^0(\vec{k}', \vec{k}; \vec{r}; z) &= t_i(\vec{k}', \vec{k}; \vec{r}; z) \\ &+ \int \frac{d^3k''}{(2\pi)^3} \frac{t_i(\vec{k}', \vec{k}''; \vec{r}; z)}{z - \dfrac{k''^2}{2\mu}} \sum_{j \neq i} T_j^0(\vec{k}'', \vec{k}; \vec{r}; z). \end{aligned} \tag{17}$$

The amplitude t_i describes the scattering of the η-meson off the i-th nucleon. It is expressed in terms of the corresponding two-body $t_{\eta N}$-matrix via

$$t_i(\vec{k}', \vec{k}; \vec{r}; z) = t_{\eta N}(\vec{k}', \vec{k}; z) \exp\left[i(\vec{k} - \vec{k}') \cdot \vec{r}_i \right],$$

where $\vec{r}_i$ is the vector from the nuclear center of mass to the i-th nucleon and can be expressed in terms of the Jacobi coordinates $\{\vec{r}\}$.

To get the nuclear wave function we used the so-called Integro–Differential Equation–Approach (IDEA) [8,9]. In this approach the wave-function of a nucleus consisting of A nucleons is decomposed in Faddeev-type components

$$\Psi(\mathbf{r}) = \sum_{i<j\leq A} \psi_{ij}(\mathbf{r}),$$

obeying

$$(h_0 - E)\psi_{ij}(\mathbf{r}) = -V(r_{ij}) \sum_{k<l\le A} \psi_{kl}(\mathbf{r}).$$

Here h_0 is the kinetic energy operator of the nucleus, and V_{ij} is the potential between nucleons i and j. These components are written as a product of a harmonic polynomial $H_{[L_m]}$ and an unknown function P

$$\psi_{ij}(\mathbf{r}) = H_{[L_m]}(\mathbf{r}) \frac{1}{\rho^{\mathcal{L}_0+1}} P(z,\rho),$$

where P depends only on the pair separation $r_{ij} = \sqrt{\rho(1+z)/2}$ and the hyperradius $\rho = \left[2/A \sum r_{ij}^2\right]^{1/2}$. Eventually, one has to solve an integro-differential equation for this function P which, for an A-boson system, reads [8,9]

$$\begin{aligned}
&\left\{\frac{\hbar^2}{m}\left[-\frac{\partial^2}{\partial\rho^2} + \frac{\mathcal{L}_0(\mathcal{L}_0+1)}{\rho^2} - \frac{4}{\rho^2}\frac{1}{W_0(z)}\frac{\partial}{\partial z}(1-z^2)W_0(z)\frac{\partial}{\partial z}\right]\right. \\
&\qquad \left. + \frac{A(A-1)}{2}V_0(\rho) - E\right\} P(z,\rho) = \\
&\qquad - \left[V\left(\rho\sqrt{\frac{1+z}{2}}\right) - V_0(\rho)\right]\left\{P(z,\rho) + \int_{-1}^{+1} f_0(z,z')P(z',\rho)dz'\right\}.
\end{aligned}$$

$V_0(\rho)$ is the hypercentral potential. The details of the method which takes into account the two-body correlations exactly can be found in Refs. [8,9].

III. RESULTS

To solve Eq. (15) we need, as an input to Eq. (17), the two–body ηN–amplitude (or the corrresponding t-matrix) and the ground state wave function Ψ_0^A of the nucleus. For $t_{\eta N}$ we employ the S-wave separable form

$$t_{\eta N}(k,k',z) = \frac{\lambda}{(k^2+\alpha^2)(z - E_0 + i\Gamma/2)(k'^2+\alpha^2)} \tag{18}$$

where E_0 and Γ are the position and width of the S_{11} resonance. Different sets of parameters α and λ are chosen such that the scattering lengths in [10] are reproduced.

For such a form, the equation for T^0 can be solved analytically. The results thus obtained are given in Table 1. The required nuclear wave functions were constructed using the Integro–differential Equation with the Malfliet-Tjon I+III (MT) nucleon–nucleon potential [11]. As can be seen there is a strong dependence of the resulting η–nucleus scattering length on the two-body input. A large ηN–scattering length generates a strong attraction of the η–meson by the three– and four–nucleon systems.

In Table 2 we present η–nucleus scattering lengths obtained by using two different sets of the nuclear wave functions, namely, the one constructed using the IDEA with MT potential

and a phenomenological one of Gaussian form chosen to reproduce the root mean square radii of the nuclei obtained via the IDEA. We see that the considerable difference found for ^2H practically vanishes when going over to ^4He.

TABLES

TABLE I. The η-nucleus scattering lengths (in fm) for 9 combinations of the range parameter α and the $a_{\eta N}$ scattering lenght [10,12,13].

	α=2.357 (fm^{-1})	α =3.316 (fm^{-1})	α =7.617 (fm^{-1})	$\alpha_{\eta N}$ (fm)
^{2}H	0.65+i0.86	0.62+i0.91	0.54+i0.97	
^{3}H	0.81+i1.90	0.66+i1.98	0.41+i2.00	0.27+i0.22
^{3}He	0.79+i1.90	0.64+i1.98	0.39+i2.00	
^{4}He	0.23+i3.54	-0.40+i3.43	-0.96+i2.95	
^{2}H	0.74+i0.77	0.73+i0.83	0.67+i0.91	
^{3}H	1.12+i1.82	0.99+i1.96	0.71+i2.07	0.28+i0.19
^{3}He	1.10+i1.83	0.97+i1.91	0.68+i2.07	
^{4}He	0.96+i3.99	0.06+i4.13	-0.91+i3.62	
^{2}H	1.38+i2.27	1.05+i2.45	0.57+i2.34	
^{3}H	-1.44+i4.60	-1.72+i3.76	-1.56+i3.00	0.55+i0.30
^{3}He	-1.42+i4.54	-1.68+i3.74	-1.53+i2.99	
^{4}He	-3.66+i1.78	-2.96+i1.39	-2.42+i1.25	

TABLE II. The η-nucleus scattering lengths (in fm) for different nuclear wave functions. In all cases $a_{\eta N}$=0.55+i0.30 fm.

Model	IDEA	Gaussian
^{2}H	1.05+i2.45	0.57+i2.14
^{3}H	-1.72+i3.76	-1.42+i4.32
^{3}He	-1.68+i3.74	-1.39+i4.28
^{4}He	-2.96+i1.39	-3.07+i1.48

To find at which values of the ηN–scattering length a quasi–bound state appears in the η–nucleus system, we write the $\eta - N$ scattering length in terms of two parameters g_1 and g_2

$$a_{\eta N} = (0.55g_1 + i0.30g_2)\ \text{fm}.$$

Note that $g_1 = g_2 = 1$ is just one of the choices in Table 1. These parameters are varied until the η-nucleus T-matrix exhibits a pole for $Re\,E = 0$. Preliminary results of this search with Gaussian type wave functions are given in Table 3.

TABLE III. Values of g_1 at which a quasi–bound state appears.

	$\alpha = 2.357$ (fm $^{-1}$)	$\alpha = 3.316$ (fm $^{-1}$)	$\alpha = 7.617$ (fm $^{-1}$)	g_2
d	1.610	1.613	1.547	0
	1.654	1.566	1.535	1
t	1.293	1.124	0.996	0
	1.361	1.310	1.260	1
^{3}He	1.302	1.204	1.130	0
	1.330	1.221	1.144	1
^{4}He	1.075	0.992	0.910	0
	0.955	0.911	0.899	1

It is seen that an $\eta - {}^4He$ quasi–bound state for realistic values of $a_{\eta N}$ can exist. In all other cases, the physical ηN input, has to be modified in order to be able to support a corresponding quasi–bound state.

Finally, we mention that using the auxiliary matrix T^0 instead of the full solution of Eq. (15), we obtained completely different results.

IV. CONCLUSIONS

We have investigated ηA systems with $A \leq 4$ in the framework of a few-body microscopic approach by using different two-body input parameters and nuclear bound state wave functions. The results can be summarized as follows:

- There is a remarkable increase of attraction with increasing atomic number.
- The same tendency appears for all nuclei when the ηN scattering length is increased.
- It is important in calculating ηA observables to use microscopically constructed wave functions.
- The fixed scatterer approximation, corresponding to the use of T^0 instead of T, is inadequate in describing the η–nucleus system.
- There is a strong indication that the $\eta\,{}^4He$ system can form a quasi–bound state.

REFERENCES

[1] M. G. Sapozhnikov, Preprint JINR (Dubna) E15-94-501; J. Ellis et al. CERN preprint, TH/94-7326, Geneva,1994.

[2] Q. Haider, L. C. Liu, Phys. Lett., **172 B**, 257(1986)

[3] R. Frascaria et al., Phys. Rev. C **50**, R537(1994)

[4] C. Y. Cheung, Phys. Lett., **119 B**, 47(1982)

[5] C. Wilkin, Phys. Lett., **331 B**, 276(1994)

[6] S. A. Coon and B. M. Preedons. Phys. Rev. C **33**, 605(1986)

[7] V. B. Belyaev, in *Lectures on the theory of few-body systems*, Springer-Verlag, Heidelberg, 1990

[8] M. Fabre de la Ripelle, H. Fiedeldey, and S. A. Sofianos , Phys. Rev. C **38**, 449(1988)

[9] W. Oehm, S. A. Sofianos, H. Fiedeldey, and M. Fabre de la Ripelle, Phys. Rev. C**44**, 81(1991)

[10] R. S.Bhalerao, L. C. Liu, Phys. Rev. Lett., **54**, 865(1985)

[11] R. A. Malfliet and J.A. Tjon, Nucl. Phys. **A127**, 161 (1969); Ann. Phys. (N.Y.) **61**, 425 (1970)

[12] C. Wilkin, Phys. Rev., C **47**, R938(1993)

[13] C. Bennhold, H. Tanabe, Nucl. Phys., **A 530**, 625(1991)